

Detecting Sub-eV Scale Physics by Interferometry

H. Tam and Q. Yang

Department of Physics, University of Florida, Gainesville, FL 32611

We propose an interferometry experiment for the detection of sub-eV scale particles such as axion-like particles (ALPs). A laser beam traverses a region permeated by a magnetic field, where photons are converted to ALPs via the Primakoff process, resulting in a slight power loss and phase shift. The beam is then combined with a reference beam that originates from the same source. The detection of a change in the output intensity would signal the presence of ALPs (or possibly other particles that couple to the photon in a similar way). Because only one stage of conversion is needed, the signal is of $\mathcal{O}(g_{a\gamma\gamma}^2)$, as opposed to $\mathcal{O}(g_{a\gamma\gamma}^4)$ for photon-regeneration experiments. This improvement over photon-regeneration is nullified by the presence of shot noise, which however can be reduced by the use of squeezed light. Additionally, our setup can incorporate optical delay lines or Fabry-Perot cavities, boosting the signal by a factor of n , where n is the number of times the laser beam is folded. This way, we can constrain $g_{a\gamma\gamma}$ better by yet another factor of $n^{1/2}$, as compared to the $n^{1/4}$ boost that would be achieved in photon-regeneration experiments.

The exploration of particle physics in the low-energy frontier began with the introduction of the QCD axions [2–7] to solve the strong-CP problem [1]. The properties of the axion are essentially characterized by one parameter – the energy scale at which the PQ symmetry is spontaneously broken, f_a . Using limits from astrophysics (as stellar emission of axions would heat up stars and accelerate their evolution [8–11]) and cosmology (avoiding overclosing the universe [12–16]), the value of f_a can be constrained to $10^9 < f_a < 10^{12}$ GeV (or $10^{-15}\text{GeV}^{-1} < g_{a\gamma\gamma} < 10^{-11}\text{GeV}^{-1}$) which implies that $10^{-6} < m_a < 10^{-3}$ eV. In this mass range, cold axions have the right properties and cosmological abundance to be a substantial fraction of dark matter.

ALPs are predicted to exist generically in string theory [19]. While pseudoscalar ALPs couple to photons as axions do, scalar ALPs couple to photons via a $aF_{\mu\nu}F^{\mu\nu}$ term in the Lagrangian, so they can be produced by photons whose polarization is perpendicular to the background magnetic field [20]. In general, there is no a priori relationship between their mass and couplings of ALPs; hence their parameter space is a lot less constrained compared to axions. In addition, many ultralight hidden-sector particles can also couple to photons as ALPs do, such as sub-eV hidden Higgs [21]. These particles are predicted naturally from the hidden sector of theories embedding the Standard Model. There are thus ample reasons to believe that new physics might lurk at the sub-eV scale, as well as the TeV scale, waiting to be discovered. However, unlike for the TeV scale physics, particle colliders may be not the best ways to detecting these weakly interacting sub-eV particles (WISPs). There thus a large number of non-collider experiments worldwide are currently actively searching for WISPs e.g. BFRF [25], BMV [27], ADMX [29], CAST [24], PVLAS [26], GammeV [30], CARRACK [31], ALPS(at DESY) [33], OSQAR(at CERN) [28], etc. As of today the hypothetical particles remain elusive. In this Letter, we propose a new experimental method [32] based on interferometry. Although the following discussion aims the detection of ALPs, it can be generalized to include ultra-

light hidden-sector particles that couple to photons in a similar way.

The photon-axion mixing in a magnetic field is due to the effective $aF\tilde{F}$ interaction term. If the polarization of the photon is parallel to the magnetic field, the probability of conversion η can be obtained from the cross section of this process, which was first done in [17, 18] and is given by

$$\eta_{\gamma \rightarrow a} = \frac{1}{4v_a}(g_{a\gamma\gamma}BL)^2 \left(\frac{2}{qL} \sin\left(\frac{qL}{2}\right) \right)^2, \quad (1)$$

where v_a is the velocity of the axion, B the magnetic field, L the length of the conversion region, and q the momentum transfer to the magnet. Since $m_a \ll \omega_\gamma \sim \text{eV}$, the frequency of the laser beam photons, $v_a \sim 1$, $q = m_a^2/2\omega_\gamma$. For $L \sim 10\text{m}$, $m_a \sim 10^{-6}\text{eV}$, this also implies that $qL \sim 10^{-5} \ll 1$. So (1) can be approximated by

$$\eta_{\gamma \rightarrow a} \approx \frac{1}{4}(g_{a\gamma\gamma}BL)^2. \quad (2)$$

If we use $B \sim 10\text{T}$, $L \sim 10\text{m}$, and $g_{a\gamma\gamma} \sim 10^{-15}\text{GeV}^{-1}$, the probability of photon-axion conversion is of $\mathcal{O}(10^{-26})$.

After the conversion, the amplitude A of the photon is reduced to $A - \delta A$, where

$$\delta A_{\gamma \rightarrow a} = \frac{A\eta_{\gamma \rightarrow a}}{2} \approx \frac{g_{a\gamma\gamma}^2 B^2 L^2 A}{8}. \quad (3)$$

Equation (3) is valid when $m_a \ll m_0 \equiv \sqrt{2\pi\omega_\gamma/L}$, which is about 10^{-4}eV for given L and ω_γ . If m_a is larger than m_0 the power loss effect decreases rapidly. When $m_a \gg m_0$, $\delta A_{\gamma \rightarrow a} \sim g_{a\gamma\gamma}^2 B^2 L^2 A (m_0/m_a)^4$.

We note that the discussion here is applicable to pseudoscalar ALPs, since they couple to the photon in exactly the same way. If the photon polarization is instead perpendicular to the magnetic field, the analysis is also valid for scalar ALPs, as they couple to photons via $aFF \sim \vec{B} \cdot \vec{B}$ instead.

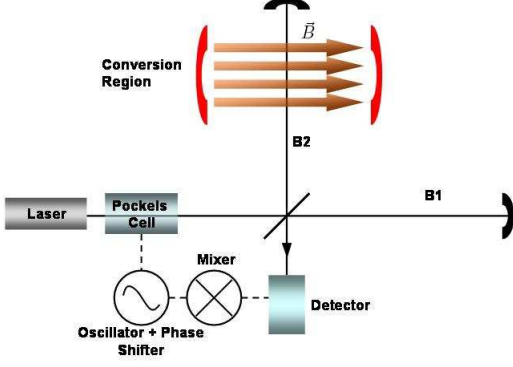


FIG. 1: Schematic diagram of our proposed experiment. A laser beam, whose amplitude is modulated by a Pockels cell, is split into two beams of equal intensity (B_1 and B_2). The beam B_2 (vertical) traverses a region permeated by a magnetic field \vec{B} . It is then recombined at the detector with the beam B_1 (horizontal), which acts as a reference.

Under weak mixing assumption, the additional phase acquired $\delta\theta$ (relative to photons that have travelled a distance L but in the absence of a magnetic field) is approximately [22]

$$\delta\theta \approx \frac{g_{a\gamma\gamma}^2 B^2 \omega_\gamma^2}{m_a^4} \left(\frac{m_a^2 L}{2\omega_\gamma} - \sin\left(\frac{m_a^2 L}{2\omega_\gamma}\right) \right). \quad (4)$$

The effect of the phase shift is negligible in comparison with $\delta A/A$ when $m_a \sim 10^{-6} \text{ eV}$. When $m_a \gg m_0$ the effect of the phase shift is comparable or even bigger than $\delta A/A$. However, as we will show, the signal due to $\delta A/A$ registered by the detector is of first order and the signal due to the phase shift registered by the detector is of second order when one uses amplitude modulation technique. Therefore as far as $\delta A/A \gg (\delta\theta)^2$, the phase shift effect is negligible.

Again, the present analysis on additional phase acquisition applies entirely to pseudoscalar ALPs. To generalize to scalar ALPs, all we need to do is to interchanging the parallel and orthogonal components of the photon relative to the magnetic field. This is expected, as $aF\tilde{F} \sim \vec{E} \cdot \vec{B}$ and $aFF \sim \vec{B} \cdot \vec{B}$, and $\vec{E}_\gamma \perp \vec{B}_\gamma$.

Even without conversion to axions, the vacuum in the presence of a magnetic field is by itself birefringent, due to loop corrections in QED (the Heisenberg-Euler term: $\frac{\alpha^2}{90m_e^4} [(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2]$) [34–36]. For $B \sim 10 \text{ T}$, $L \sim 10 \text{ m}$, $\omega \sim \text{eV}$, the QED effect of the phase shift is of $\mathcal{O}(10^{-14})$ so it is registered by the detector of order $\mathcal{O}(10^{-28})$ which is negligible.

In our proposed experiment, a laser beam first enters a Pockels cell (with a polarizer behind) to modulate its amplitude (the purpose of the modulation will be explained below). Subsequently, it is divided by a beamsplitter into two beams (which we label B_1 and B_2 in Figure 1) with equal intensity. B_2 is essentially the laser beam used

in the first half of the photon-regeneration experiment [23]: it passes through a region permeated by a constant magnetic field, where a small fraction of the photons are converted into axions which carry energy away from the beam, according to (2). For simplicity, we will consider here that the carrier of the modulated beam (both B_1 and B_2) is linearly polarized in the direction of the magnetic field (For the detection of scalar ALPs, the polarization should be perpendicular to the magnetic field instead). The two beams are then recombined at the detector, and in the presence of a conversion, the slight amplitude reduction and phase shift would lead to interference, which can be detected.

The length of the path traversed by beam B_1 is by design slightly different from that by B_2 , so that at the detector the two beams would be out of phase by π if the magnetic field has been absent. Operationally, this can be achieved by adjusting one of the path lengths until destructive interference is observed at the detector when the magnetic field is turned off. Hence, in the absence of the sidebands, the two beams would interfere destructively at the detector. The purpose for this arrangement is to reduce the background, thereby enhancing the signal-to-noise ratio and minimizing shot noise.

Let the path lengths of the two arms be L_x and L_y (corresponding to beams B_1 and B_2), and that the state of the laser after passing through the Pockels cell can be described by

$$\vec{E}_{in} = \vec{E}_0(1 + \beta \sin \omega_m t)e^{i\omega t}, \quad (5)$$

where β is a constant, \vec{E}_0 the initial electric field at $t = 0$, and ω is the frequency of the laser. The amplitude is modulated at a frequency ω_m . This can be recast as

$$\vec{E}_{in} = \vec{E}_0 \left(e^{i\omega t} + \frac{\beta}{2i} e^{i(\omega+\omega_m)t} - \frac{\beta}{2i} e^{i(\omega-\omega_m)t} \right), \quad (6)$$

where the first term is referred to as the “carrier”, and the latter two as “sidebands”.

The state of the carrier after recombination at the detector is given by

$$\begin{aligned} \vec{E}_{carrier} &= -\frac{\vec{E}_0}{2} e^{i(\omega t + 2kL)} \\ &\times \left[2i \sin k\Delta L - \left(\frac{\delta A}{A} + i\delta\theta \right) e^{-ik\Delta L} \right], \end{aligned} \quad (7)$$

where $k = \omega/c$ is the wavenumber of the laser photons, $A = |\vec{E}_0|$, $\Delta L = L_x - L_y$ is the length difference between the two arms, and $L = (L_x + L_y)/2$ is the average. As mentioned, we will choose $k\Delta L = \pi$, so that the detector operates at a dark fringe, in order to eliminate the background signal. This leads to

$$\vec{E}_{carrier} = \frac{e^{i(\omega t + 2kL)}}{2} \left(\frac{\delta A}{A} + i\delta\theta \right) \vec{E}_0. \quad (8)$$

Note that without the aid of the sidebands, this would be the entire signal. While the background is eliminated, the

intensity ($\sim \vec{E}^2$) is of $\mathcal{O}(g_{a\gamma\gamma}^4)$. This loss in sensitivity, as we will see, can be recovered by using the sidebands.

Meanwhile, the sidebands (second and third terms of (6)) are described by

$$\vec{E}_{\pm} = \vec{E}_0 \beta e^{i(\omega t + 2kL)} e^{\pm i(\omega_m t + 2\omega_m L/c)} \times \left[\sin \frac{\omega_m \Delta L}{c} \mp i \left(\frac{\delta A}{A} + i\delta\theta \right) \frac{e^{\mp i\omega_m \Delta L/c}}{2} \right], \quad (9)$$

where the subscripts $+$ and $-$ denote respectively the sideband components of frequency $\omega + \omega_m$ and $\omega - \omega_m$.

If we set $\omega_m \approx \pi c/2\Delta L$, the total electric field at the detector is obtained by adding that of the carrier and sidebands:

$$\vec{E} = \vec{E}_0 e^{i(\omega t + 2kL)} \left(\frac{1}{2} \left(\frac{\delta A}{A} + i\delta\theta \right) + \beta \left(2 - \left(\frac{\delta A}{A} + i\delta\theta \right) \right) \cos \left[\omega_m t + \frac{2\omega_m L}{c} \right] \right) \quad (10)$$

Note that this particular value of ω_m is chosen to maximize the signal. Since $\omega_m \rightarrow n\omega_m$ and $k\Delta L \rightarrow n\pi$ (for n an odd integer) are equally valid choices, the experimenter has much freedom in choosing a suitable value for ω_m that is experimentally feasible.

Hence, the power P that falls on the detector is

$$P = P_{in} \left\{ \frac{(\delta A/A)^2 + \delta\theta^2}{4} + \frac{\beta^2(4 - 4\frac{\delta A}{A} + \frac{\delta A^2}{A^2} + \delta\theta^2)}{2} + \beta \left(2\frac{\delta A}{A} - \frac{\delta A^2}{A^2} + \frac{\delta\theta^2}{2} \right) \cos \left[\omega_m \left(t + \frac{2L}{c} \right) \right] + \frac{\beta^2(4 - 4\frac{\delta A}{A} + \frac{\delta A^2}{A^2} + \delta\theta^2)}{2} \cos \left[2\omega_m \left(t + \frac{2L}{c} \right) \right] \right\} \quad (11)$$

Thus the power has a dc component (first line), and two ac components with frequencies ω_m and $2\omega_m$. If we multiply this with the oscillator voltage that drives the Pockels cell (plus an appropriate phase shift) via a mixer, we can extract the component of frequency ω_m . Neglecting the second-order contributions, the time-averaged output power of the mixer is given by

$$P_{out} = \frac{1}{T} \int_T 2P_{in} \beta \mathcal{G} \left(\frac{\delta A}{A} \right) \cos^2(\omega_m t) \quad (12)$$

$$= \frac{P_{in} \beta \mathcal{G} \delta A}{A} \quad (13)$$

where \mathcal{G} is the gain of the detector and T is taken to be sufficiently long to ensure that the time-averaging is accurate. Hence, the output signal is proportional to $g_{a\gamma\gamma}^2$ for axions or ALPs.

In this analysis we choose to modulate the amplitude, rather than the phase, of the photons so the result will not be spoiled by the QED effect. In principle, we could instead modulate the phase, in which case the change in intensity registered by the detector would be primarily a

consequence of the phase shift instead of the amplitude reduction. The corresponding analysis is highly analogous and will not be repeated here. The major difference is that the coefficients for the sidebands in (6), $\beta/2i$, are replaced approximately by $J_1(\beta)$, the first-order Bessel function of the first kind (higher harmonics now are also present, but are negligible). Since $J_1(\beta)$ are real, our earlier analysis would work if $\delta A/A$ is replaced by $i\delta\theta$, which is purely imaginary. This can be implemented by manipulating polarizers adjacent to the Pockels cell. Thus by switching between phase and amplitude modulation, we can infer information on both the amplitude reduction and phase shift. For the experiments mainly interested in measuring the QED effect, the phase modulation should be employed.

Despite the improvement in signal size, the use of interferometers is inevitably accompanied by the presence of shot noise. This limits the resolution of the interferometer therefore reducing the sensitivity to $g_{a\gamma\gamma}$ in our set up.

For a laser beam consisting of N incoming photons, we expect the shot noise in our setup to have a magnitude of \sqrt{N} due to Poisson statistics. The signal-to-noise ratio is thus reduced to $(g_{a\gamma\gamma} BL)^2 N / \sqrt{N}$. In the case of a non-detection, this allows us to constrain the axion-photon coupling to $g_{a\gamma\gamma, max} < (BL)^{-1} N^{-1/4}$, which is what can be achieved by conventional photon-regeneration experiments. (In photon-regeneration experiments, the signal is much smaller, of $\mathcal{O}(g_{a\gamma\gamma}^4 N)$, so dark count rate can be a problem.)

Our setup admits a straightforward implementation of squeezed light using standard optical techniques, which can help reduce shot noise. Using interferometry, in principle, is a different realization of the polarimetry experiment that measures birefringence and dichroism. However, in the polarimetry the dominant noise is the intrinsic birefringence of the optical devices. In interferometry, the intrinsic noises is dominated by the photon counting error (shot noise). Shot noise can be viewed as the beating of the input laser with the vacuum fluctuations entering the other side of the beam splitter. The conception of reducing shot noise by injecting squeezed light is first suggested by [37]. Let us give a brief summary in the following. The coherent state $|\alpha\rangle$ is described by the unitary displacement operator: $|\alpha\rangle = D(\alpha)|0\rangle = \exp(\alpha a^\dagger - \alpha^* a)|0\rangle$, where a^\dagger and a are creation and annihilation operators of photons and α is a complex number. The photon number operator is $N = a^\dagger a$ and one finds: $\bar{N} = |\alpha|^2$, $\Delta N = |\alpha|$ for the coherent state. A squeezed state is described as $|\alpha, \zeta\rangle = D(\alpha)S(\zeta)|0\rangle$, where $\zeta = re^{i\theta}$ is a complex number and $S = \exp[1/2(\zeta^* a^2 - \zeta(a^\dagger)^2)]$. For the squeezed state one finds: $\bar{N} = |\alpha|^2 + \sinh^2 r$ and $(\Delta N)^2 = |\alpha \cosh r - \alpha^* e^{i\theta} \sinh r|^2 + 2 \cosh^2 r \sinh^2 r$. Let mode 1⁺ denote electromagnetic field incident from the laser side of the beam splitter and mode 2⁺ denote electromagnetic field incident from the other side of beam splitter. By using an ordinary laser $|\alpha, 0\rangle$ in

one side of the beam splitter and injecting squeezed light $|0, \zeta\rangle$ from the other side of beam splitter we have the state: $|\phi\rangle = S_2(\zeta)D_1(\alpha)|0\rangle$. The photons received by an ideal photo-detector in one output port then have the property: $\bar{N} = \alpha^2 \sin^2(\phi/2) + \cos^2(\phi/2) \sinh^2 r$ and $\Delta N^2 = \alpha^2 \sin^4(\phi/2) + 2\cos^4(\phi/2) \cosh^2 r \sinh^2 r + \sin^2(\phi/2) \cos^2(\phi/2) (\alpha^2 e^{-2r} + \sinh^2 r)$, where ϕ is the phase difference between the two arms of the interferometer. We see that if one operates near a dark fringe, $\bar{N} = \sinh^2 r$ and $(\Delta N)^2 = 2\cosh^2 r \sinh^2 r$ which can be arbitrarily small in theory. Implementations of squeezed light together with using power recycling and sidebands are demonstrated by [38] and later a 10dB shot noise reduction is achieved [39]. A 10dB suppression of shot noise can result in a $10^{1/2}$ improvement of the constraint to $g_{a\gamma\gamma}$.

To further boost the sensitivity, we can incorporate in our setup optical delay lines or Fabry-Perot cavities to enhance the signal by a factor of n , where n is the number of times the laser beam is folded. The resultant improvement in our ability to constrain $g_{a\gamma\gamma}$ is of order $n^{1/2} \sim 1000$ v.s. $n^{1/4} \sim 10^{1.5}$ in photon regeneration experiment. Combined, the use of squeezed light and optical delay lines results in a gain in the sensitivity to

$g_{a\gamma\gamma}$ of 10^2 over a power recycled photon regeneration experiment for current techniques.

If we use $n \sim 10^6$, $B \sim 10\text{T}$, $L \sim 10\text{m}$ with a 10W ($\lambda = 1\mu\text{m}$) laser, after 240 hours running, the experiment can exclude ALPs with $g_{a\gamma\gamma} > 0.9 \times 10^{-11} \text{GeV}^{-1}$ to 5σ significance. If one also employs squeezed-light laser which improves signal-to-noise ratio by 10dB with similar setup, the exclusion limit can reach $g_{a\gamma\gamma} \sim 3 \times 10^{-12} \text{GeV}^{-1}$.

While we have as our principal aim the detection of ALPs, our design is theoretically applicable to any particle with a two photon vertex, so that mixing in the presence of an external magnetic field is permitted. Given the possibility that more than one such particle exists, it is important to identify what the photons have converted into. We suggest two methods that can help shed light on this issue. First, we could repeat the experiment by modulating the phase instead of the amplitude of the laser, as this would reveal information about the phase shift as well. Secondly, scalar and pseudoscalar ALPs can be distinguished by modifying the polarization of the laser. Conversion can only occur if the polarization is parallel (perpendicular) to the external magnetic field for pseudoscalar (scalar) ALPs.

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